Interaction between Variable Chaplygin gas and Tachyonic Matter

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Abstract

We consider a model of varying Chaplyagin gas interacting with a Tachyonic matter in framework of GR. As a varying Chaplygin gas we consider the model considered in [4]. Goal of the letter is to reconstruct and investigate the potential $V(\phi)$ and ω_{tot} . To achieve the final goal analytical and numerical techniques are used depends on necessity. Graphical analysis is performed and presented.

Introduction

Explanation of experimental data concerning to the nature of accelerated expansion of the Universe in GR could be achieved by so-called Dark Energy: an exotic and mysterious component of the Universe, with negative pressure and positive energy density implying negative EoS parameter $\omega < 0$. The simplest model of Dark Energy is a cosmological constant $\omega_{\Lambda}=-1$ introduced by Einstein, but it suffers with the problem known as cosmological coincidence problem: why are we living in an epoch in which the densities of the Dark Energy and matter are comparable? One of the ways to solve the cosmological coincidence problem is to consider the interaction between components. In literature different forms of interaction term were considered, started from $Q = 3Hb\rho_m$, $Q = 3Hb\rho_{de}$, $Q = 3Hb\rho_{tot}$, where b is a coupling constant, to time dependent forms $Q = \gamma \dot{\rho}_m$, $Q = \gamma \dot{\rho}_{de}$, $Q = \gamma \dot{\rho}_{tot}$, and interaction of the general form $Q = 3Hb\gamma\rho_i + \gamma\dot{\rho}_i$, where $i = \{m, de, tot\}$. Generally interaction could be considered as a function of energy densities and their derivatives: $Q(\rho, \dot{\rho}, \ldots)$. To make everything working only should be considered that $[Q] = \frac{[energy\ density]}{time}$ and assume that unit $time^{-1}$ could be contribution from Hubble parameter, which gives simple forms of the interactions provided above. Before going further, we would like to notice the fact, that there is not any fundamental explanation concerning to the nature and mathematical forms of the interaction considered in literature over the years. Here we should mention that everything is done on phenomenological level. These kind of interactions are either positive or negative and can not change sign. Hereafter we introduce a type of interaction, called sign-changeable [1],[2], of the following form

$$Q = q(\gamma \dot{\rho} + 3bH\rho),\tag{1}$$

where γ and b are both dimensionless constants, ρ is the energy density and could be ρ_m , ρ_{de} or ρ_{tot} , q is the deceleration parameter

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a}.\tag{2}$$

H is the Hubble parameter and a(t) is a scale factor. Concluding, we would like to add, that by this way we import more information about geometry of the Universe into the interaction term.

Scalar fields thought to be one of the ways of describing dark energy. For recent investigation consider a scalar field given by relativistic Lagrangian and known as Tachyonic field [8].

$$L = -V(\phi)\sqrt{1 - \partial_{\mu}\phi\partial^{\mu}\phi}.$$
 (3)

Energy density and pressure for this model are

$$\rho_{TF} = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}.\tag{4}$$

and

$$P_{TF} = -V(\phi)\sqrt{1 - \dot{\phi}^2}.$$
 (5)

Other model is a Chaplygin Gas [3] with EoS

$$P_{cg} = -\frac{A}{\rho_{cg}}. (6)$$

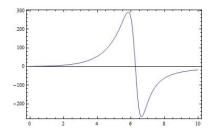


Figure 1: The variation of \ddot{a} against t

Such equation of state leads to a component, which behaves as dust at early stage and as cosmological constant at later stage. In this letter we are interested in Variable Chaplygin gas [4]-[7] where A depends on scale factor: A(a), as follows

$$A(a) = A_0 a^{-m}, (7)$$

where $A_0 > 0$ and m are constants. The purpose of this letter is to consider a mixture of Variable Chaplyagin gas interacting with Tachyonic matter, where as interaction between components will be considered separately $Q = 3Hb\rho_{cg} + \gamma\dot{\rho}_{cg}$ and $Q = q(3Hb\rho_{cg} + \gamma\dot{\rho}_{cg})$. The mixture of our consideration will be described by ρ_{tot} and P_{tot} given by

$$\rho_{tot} = \rho_{TF} + \rho_{cg},\tag{8}$$

and

$$P_{tot} = P_{TF} + P_{cq}. (9)$$

Accelerating Universe will be described by $a(t) = t^n$, with n > 1, $a(t) = a_0 t^m \exp[\alpha t]$ and $a(t) = a_0 t^n + b_0 \exp[\alpha t]$ scale factors. Two Last scale factors correspond to different forms of the Universe in quasi-exponential phase [9]. We also interested in performing the analysis for a scale factor obtained in [10] and having \ddot{a} profile presented in Fig.1.

Paper organized as follow: After Introduction, in next sections the following will be happen: Field equations and reveal of interaction appearance will be given, then analysis of $V(\phi)$ and ω_{tot} for Tachyonic matter presented in order. Numerical techniques are used where they are needed.

Field Equations

Field equations that governs our model read as

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R^{\alpha}_{\alpha} = T^{\mu\nu},\tag{10}$$

and with FRW metric (the metric of a spatially flat homogeneous and isotropic universe)

$$ds^{2} = dt^{2} - a(t)^{2} \left(dr^{2} + r^{2} d\theta^{2} + r^{2} \sin^{2} \theta d\phi^{2} \right).$$
 (11)

they reduced to

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{\rho_{tot}}{3}. (12)$$

$$-\frac{\ddot{a}}{a} = \frac{1}{6}(\rho_{tot} + P_{tot}). \tag{13}$$

Bianchi identities or energy conservation condition reads as

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + P_{tot}) = 0. \tag{14}$$

To introduce an interaction between dark energy and matter (14) splits into two following equations

$$\dot{\rho}_{cg} + 3H(\rho_{cg} + P_{cg}) = Q,\tag{15}$$

and

$$\dot{\rho}_{TF} + 3H(\rho_{TF} + P_{TF}) = -Q. \tag{16}$$

It could be understood by this way, that as there is an interaction between components, therefore there is not energy conservation for the components separately, though for the whole mixture the energy conservation is hold. This approach could work as long as we are working without knowing the actual nature of the dark energy and dark matter as well as the nature of interaction. This approach at least from mathematical point of view is correct.

Interaction $Q = 3Hb\rho_{cq} + \gamma \dot{\rho}_{cq}$

In this section our attention will be payed to analyze potential of the Tachyonic matter and obtain real properties of it and at the same time investigate ω_{tot} in case of a set of the parameters. We will start with a scale factor of the form $a(t) = t^n$ with n > 1 and finish investigation with scale factor known as scale factor of the Universe in quasi-exponential phase.

Scale Factor $a = t^n$

With this form of scale factor the solution of (15) with interaction $Q=3Hb\rho_m+\gamma\dot{\rho}_m$ reads as

$$\rho_{cg} = \frac{\sqrt{-6A_0(t^n)^{-m} - \rho_0(6(-1+b) - m(-1+\gamma))t^{-\frac{6(-1+b)n}{-1+\gamma}}}}{\sqrt{6(-1+b) - m(-1+\gamma)}}.$$
(17)

The energy density for the Tachyonic field we can obtain using (17) and (12)

$$\rho_{TF} = 3\frac{n^2}{t^2} - \rho_{cg}.\tag{18}$$

Energy densities for each component allow to determine the pressures. For the pressure of Chaplygin gas we use (6) and (17) and for the pressure of Tachyonic matter from (16) we will have

$$P_{TF} = \frac{-Q - \dot{\rho}_{TF}}{3H} - \rho_{TF}.\tag{19}$$

Taking into account that EoS parameter of Tachyonic matter and filed ϕ related to each other by the following expression $\omega_{TF} = \frac{P_{TF}}{\rho_{TF}} = -1 + \dot{\phi}^2$, for field we have

$$\phi = \int \sqrt{1 + \omega_{TF}} \ dt. \tag{20}$$

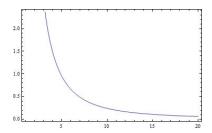


Figure 2: The variation of V against t, Interaction: $Q=3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}$, Parameters: $n=3,\ m=5,\ \rho_0=1,\ A_0=1,\ a_0=1,\ b=0.7,\ \gamma=0.5,\ \alpha=0.5$

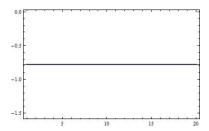


Figure 3: The variation of ω_{tot} against t, Interaction: $Q = 3Hb\rho_{cg} + \gamma\dot{\rho}_{cg}$, Parameters: $n = 3, m = 5, \rho_0 = 1, A_0 = 1, a_0 = 1, b = 0.7, \gamma = 0.5, \alpha = 0.5$

Potential can be recovered analytically by

$$V(\phi) = \sqrt{-\rho_{TF} P_{TF}}. (21)$$

" – " under the square root, will not worry us, because $P_{TF} < 0$. The other parameter of our interest is ω_{tot} defined as

$$\omega_{tot} = \frac{P_{TF} + P_{cg}}{\rho_{TF} + \rho_{cg}}. (22)$$

In coming sections we will present graphical analysis of evolution of the potential V and ω_{tot} over time. Analysis reveals, that $V \to 0$ over time, which means that real properties of Tachyonic field are possible to obtain and $\omega_{tot} > -1$ reveals quintessence-like behavior during whole evolution of the Universe: from early epoch to late stage (see fig. 2 and 3). Mathematics presented in this section will be used through to the whole article for further investigations.

Scale Factor $a = a_0 t^n \exp[\alpha t]$

Hubble parameter corresponding to scale factor $a=a_0t^n\exp[\alpha t]$ characterizing the Universe in quasi-exponential phase reads as

$$H = nt^{-1} + \alpha. (23)$$

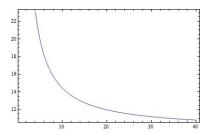


Figure 4: The variation of V against t, Interaction: $Q=3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}$, Parameters: $n=4,\ m=10,\ \rho_0=1,\ A_0=1,\ a_0=1,\ b=0.05,\ \gamma=0.3,\ \alpha=1.8$

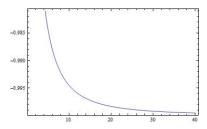


Figure 5: The variation of ω_{tot} against t, Interaction: $Q=3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}$, Parameters: $n=4,\ m=10,\ \rho_0=1,\ A_0=1,\ b=0.05,\ \gamma=0.3,\ \alpha=1.8$

Analysis, as in case of $a(t) = t^n$, reveals, that $V \to 0$ over time, which means that the real properties of Tachyonic field were possible to obtain and $\omega_{tot} > -1$ reveals quintessence-like behavior during whole evolution of the Universe: from early epoch to late stage (see fig 4 and 5).

Scale Factor $a(t) = a_0 t^n + b_0 \exp[\alpha t]$

Another scale factor $a(t)=a_0t^n+b_0\exp[\alpha t]$ characterizing the Universe in quasi-exponential phase for Hubble parameter gives us the following

$$H = \frac{a_0 n t^{-1+n} + \alpha b_0 e^{\alpha t}}{a_0 e^{\alpha t} + a_0 t^n}.$$
 (24)

For this case numerical analysis of potential $V(\phi)$ and ω_{tot} reveals the following (see fig 6 and 7). During whole evolution of the Universe: from early epoch to late stage $\omega_{tot} > -1$ meaning that mixture indicates quintessence-like behavior in case of $a(t) = a_0 t^n + b_0 \exp[\alpha t]$.

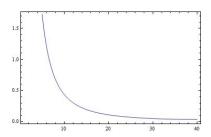


Figure 6: The variation of V against t, Interaction: $Q=3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}$, Parameters: $n=4,\ m=5,\ \rho_0=1,\ A_0=1,\ a_0=1.5,\ b_0=1,\ b=0.8,\ \gamma=0.7,\ \alpha=0.3$

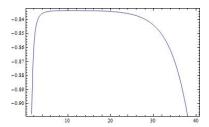


Figure 7: The variation of ω_{tot} against t, Interaction: $Q=3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}$, Parameters: $n=4,\ m=5,\ \rho_0=1,\ A_0=1,\ a_0=1.5,\ b_0=1,\ b=0.8,\ \gamma=0.7,\ \alpha=0.3$

Interaction $Q = q(3Hb\rho_m + \gamma \dot{\rho}_m)$

In this section we present investigation corresponding to a sign-changeable interaction between Chaplygin gas and Tachyonic matter. After analysis, following exactly the mathematics of the previous section, we reveal that for all three cases of the scale factor we are able to obtain desirable properties of Tachyonic field: $V \to 0$ over time (see fig 8, 10 and 12). Fixing parameters for both cases of the model in order to satisfy this condition we observe that $\omega_{tot} > -1$ during whole evolution of the Universe, which means that we have quintessence-like behavior for the model (see fig 9, 11 and 13).

Scale factor from [10]

Analysis for both type of interactions considered in previous sections, concerning to this kind of scale factor reveals, that a set of values (values known from different experiments) for the set of parameters of the model allowing as to obtain satisfactory condition $V \to 0$ over time for the potential of Tachyonic field, provide a behavior $\omega_{tot} \geq 0$. For some cases we obtain $\omega_{tot} > -1$ but very rapidly it again becomes positive and stay positive during the evolution.

Discussion

Analysis of this article reveals that the model of Chaplygin gas interacting with Tachyonic matter with both type of interactions $Q = 3Hb\rho_{cg} + \gamma\dot{\rho}_{cg}$ and $Q = q(3Hb\rho_{cg} + \gamma\dot{\rho}_{cg})$ indicates quintessence-like behavior during whole evolution of the Universe. Type of the behavior was the same for three scale factors: $a(t) = t^n$, $a(t) = a_0t^n \exp[\alpha t]$ and $a(t) = a_0t^n + b_0 \exp[\alpha t]$. It is obvious that the first scale factor is special case of two other ones. In [11] interaction between Chaplygin gas and Tachyonic matter was considered in case $a(t) = t^n$, which made motivation of this work. EoS parameter of considered mixture in Universe with a scale factor of [10] is positive during hole evolution, for some values of parameters $\omega_{tot} > -1$, which disappear very rapidly and EoS parameter become positive.

Acknowledgments

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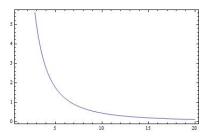


Figure 8: The variation of V against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg})$, Parameters: $n=4,\ m=2,\ \rho_0=1,\ A_0=1,\ a_0=1,\ b_0=0.5,\ b=0.9,\ \gamma=0.6,\ \alpha=0.3$

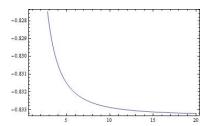


Figure 9: The variation of ω_{tot} against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg})$, Parameters: $n=4,\ m=2,\ \rho_0=1,\ A_0=1,\ a_0=1,\ b_0=0.5,\ b=0.9,\ \gamma=0.6,\ \alpha=0.3$

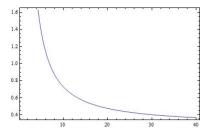


Figure 10: The variation of V against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg}),$ Parameters: $n=2,\ m=3,\ \rho_0=1,\ A_0=1,\ a_0=1.5,\ b_0=0.5,\ b=0.2,\ \gamma=0.6,$ $\alpha=0.3$

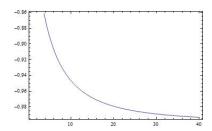


Figure 11: The variation of ω_{tot} against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg})$, Parameters: $n=2,\ m=3,\ \rho_0=1,\ A_0=1,\ a_0=1.5,\ b_0=0.5,\ b=0.2,\ \gamma=0.6,\ \alpha=0.3$

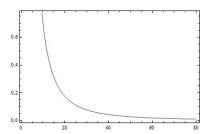


Figure 12: The variation of V against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg})$, Parameters: $n=5,\ m=2,\ \rho_0=1,\ A_0=1,\ a_0=0.5,\ b_0=1.5,\ b=0.6,\ \gamma=0.3,\ \alpha=0.5$

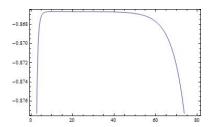


Figure 13: The variation of ω_{tot} against t, Interaction: $Q=q(3Hb\rho_{cg}+\gamma\dot{\rho}_{cg})$, Parameters: $n=5,\ m=2,\ \rho_0=1,\ A_0=1,\ a_0=0.5,\ b_0=1.5,\ b=0.6,\ \gamma=0.3,\ \alpha=0.5$